Geometric Convergence of Elliptical Slice Sampling

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Setting and Elliptical Slice Sampler

Setting: For reference measure $\mu_0 = \mathcal{N}(0, C)$ sample w.r.t.

$$\mu(A) = \frac{\int_A \varrho(x)\mu_0(\mathrm{d}x)}{\int_{\mathbb{R}^d} \varrho(x)\mu_0(\mathrm{d}x)}, \qquad A \subseteq \mathbb{R}^d.$$

Elliptical Slice Sampler:

- generates a Markov chain $\{X_n\}$ for approximate sampling of μ
- introduced by Murray, Adams, MacKay in 2010
- **no** convergence theory $\longrightarrow$ this paper

Convergence result

Assumptions:

- ϱ is bounded away from 0 and ∞ on any compact set of $\mathbb{R}^d$
- $\exists \alpha > 0, R > 0$, such that $\forall \|x\| > R$ holds

$$\left\{y \in \mathbb{R}^d : \|y\| \leq \alpha \|x\|\right\} \subseteq \left\{y \in \mathbb{R}^d : \varrho(y) \geq \varrho(x)\right\}$$

Theorem (geometric ergodicity)

$\exists K > 0, \exists \gamma \in (0, 1)$, such that

$$\|\mathbb{P}(X_n \in \cdot \mid X_1 = x) - \mu(\cdot)\|_{\text{tv}} \leq K \gamma^n (1 + \|x\|), \qquad \forall x \in \mathbb{R}^d.$$

Applicable to:

- Gaussian setting
- Logistic regression setting
- ...

Thank you for your attention.